PIT-TTC PD FRAMEWORKS FOR CREDIT RISK CLASSIFICATION SYSTEMS

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Alexander Petrov, PhD, Executive Advisor, Team Lead, Risk Models, Nordea group, is leading and responsible for different risk models development and implementation projects in the Nordea group, including IRB, IFRS9, ICAAP, stress testing, etc. Member and Nordea representative in Global Credit Data Methodological Committee. Affiliated researcher and guest lecturer at Stockholm Business School, Stockholm University.

Petrov has a broad experience in the mathematical modelling with a special interest in risk management and banking/insurance applications, having several publications in international journals and 18+ years experience in the risk models development. Previously Petrov managed several overall implementation of the Basel II IRB approach projects in the banks on emerging markets in Russia, Ukraine, Baltics as well as in the Swedbank Group Risk Control Department, he was responsible for all modelling/quantitative issues, as well as for implementation of Swedbank group risk management standards in the local BUs in the emerging markets.

Another areas of Petrov professional experience were actuarial models development and Solvency 2 internal models development during his employment in RSA Scandinavia, as well as mathematical algorithms development for DSP and queuing theory development for telecom applications when he was working for Ericsson.

Petrov obtained his PhD in theoretical physics from Uppsala University, Sweden.

AGENDA

► Introduction: Why PIT-TTC framework?

► Methodology for PIT-TTC PD adjustment based on Carlehed, Petrov: "A methodology for point-in-time–through-the-cycle probability of default decomposition in risk classification systems",
Journal of Risk Model Validation, 2012

► Application and usage of PIT-TTC methodology

Introduction: Why PIT-TTC PD Framework?

DEFINITIONS

| Concept	Definition
Probability of Default (PD)	The probability that a counterparty will default in a 12 month period
Point-In-Time (PIT) PD	The probability that a counterparty will default in the next 12 months, given today's information of the counterparty, the economic cycle, etc. Forecast of Actual Default Frequency (ADF)
Through-The- Cycle (TTC) PD	The probability that a counterparty will default over an arbitrary 12 months period, unconditional of the states of economic cycle

Why through-the-cycle PD adjustment?

According to the Basel IRB approach…	In real life…
For calculation of capital buffer against unexpected losses, the through-the-cycle PD (unconditional of the states of economic cycle) should be used in the RWA formulas.	All rating models are so-called hybrid models, i.e. models that vary with the economic cycle to a certain extent, but not fully.

Why through-the-cycle PD adjustment?
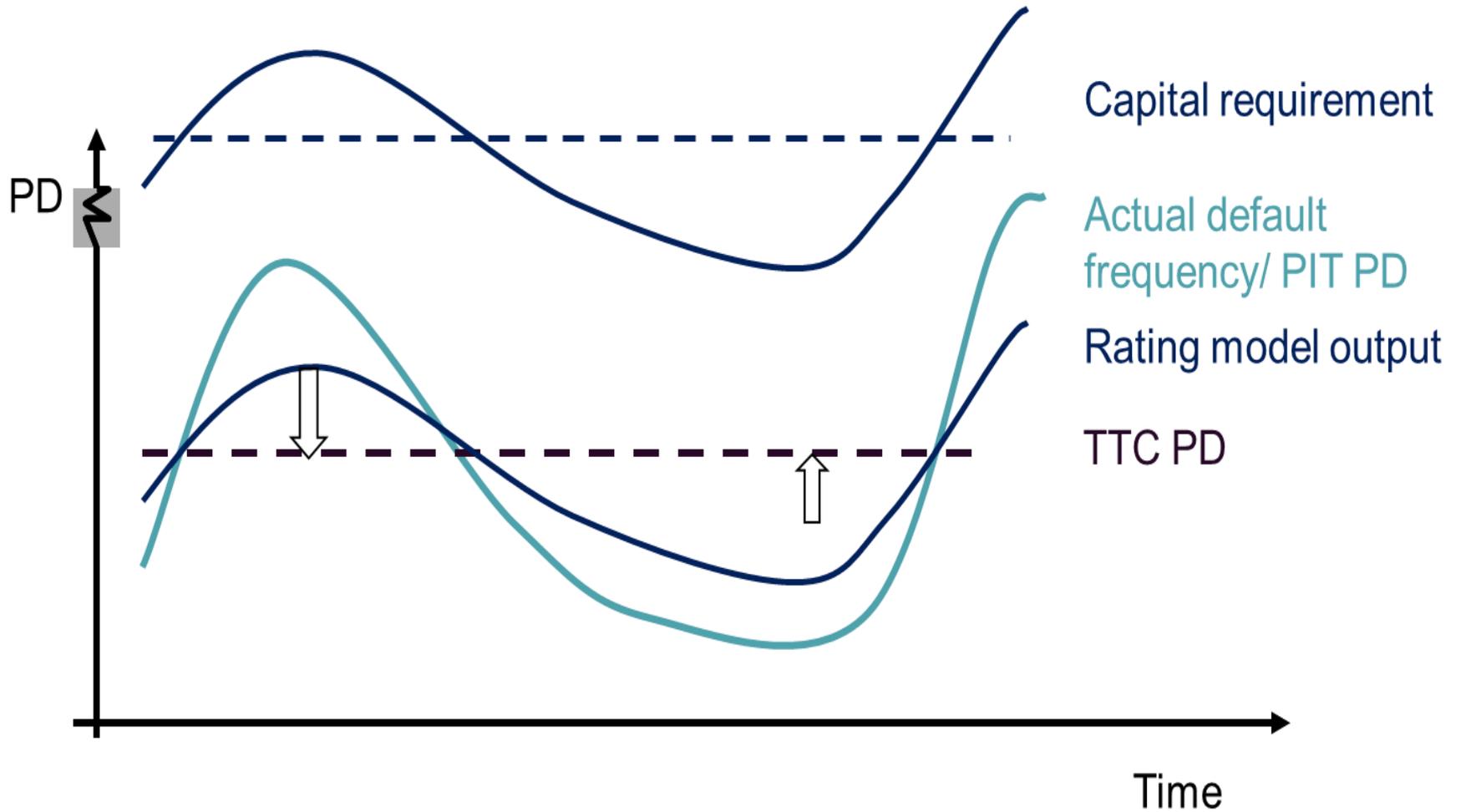
Distinct applications for PIT and TTC PDs inside a bank:

- PIT: IFRS9, ICAAP, Stress testing, PD model validation, pricing of short-term products;

- TTC: Capital requirements, pricing of long-term products;

- Both: Other pricing, performance measures, risk appetite.

Need for internal consistency:

- Different parts of Bank typically produce their own PDs that do not speak to each other, based on different models and perhaps even different datasets;

- IFRS9, Stress testing and ICAAP all use their own PITs;

- Capital requirements are based on Hybrid PDs;

- Validation, customer selection and loan pricing use the same Hybrid PD;

Basel formula requires TTC:

- Although regulation leaves a lot of leeway, stricktly speaking, it is TTC that should be used in Basel risk-weight formula;

- The use of Hybrid PD brings pro-cyclicality in capital requirements and risk to macro stability, something that regulators want to avoid;

- MoC is to be applied to TTC PDs. Using Hybrid for MoC calculations may result in over-estimation.

Validation needs PIT

- PD back-testing must compare quantities with same PIT-ness. This means, e.g. PIT default rates vs. PIT PD;

- Using Hybrids makes it hard to interpret a PD-ADF deviation, especially if the Hybrid's PIT-ness is unknown;

- Uninformative Hybrid-based validation results increase the risk of regulatory santctions

Hybrids hurt business

- Short-term products must use PIT PD; long-term – TTC. The one-size-fits-all approach of Hybrid is a handicap leading to inefficient customer selection;

- Hybrid PD = pro-cyclical REA = pro-cyclical lending pattern = pro-cyclical profits = depressed stock price.

Methodology for PIT-TTC adjustment based on Carlehed & Petrov: A methodology for point-in-time–through-the-cycle probability of default decomposition in risk classification systems, Journal of Risk Model validation, 2012

Estimation of the economic cycle by using external data

Individual obligor *i* <u></u>

- Standard Merton assumptions are used (same as in Basel II):

$$X_i = \sqrt{\rho}Z + \sqrt{1 - \rho}\varepsilon_i \text{ (proxy for the assets of counterparty } i),$$

  - Z systemic variable, standard normal
  - ρ asset correlation
  - ε_i idiosyncratic risk of counterparty i, standard normal
  - B_i (proxy for the debt of counterparty i)
- The counterparty *i* defaults when $X_i < B_i$, which leads to

$$\mathrm{PD}_i = \Phi\left[(B_i - \sqrt{\rho}Z)/\sqrt{1 - \rho}\right]$$

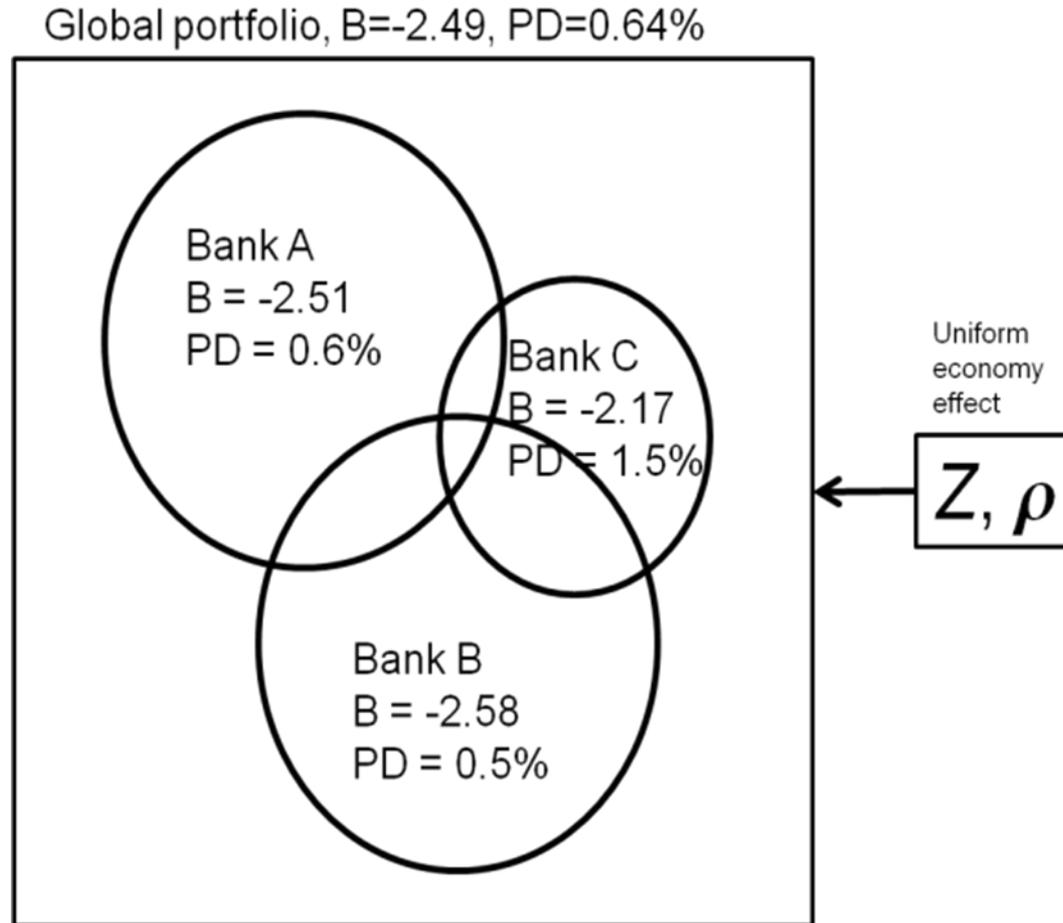# Estimation of the economic cycle by using external data

Portfolio

- Bank's portfolio is a part of the global portfolio

- The same average "economy effect" on each obligor in the global portfolio, dependent on the sector specific variable Z and ρ

- The observed default rate in global portfolio will be:

$$\mathrm{ADF} = \Phi\left[(B - \sqrt{\rho}Z)/\sqrt{1-\rho}\right],$$

where B can be interpreted as belonging to average or "central tendency" client of the global portfolio

Bank's portfolio and the global portfolio

Global portfolio, B=-2.49, PD=0.64%

Bank A
B = -2.51
PD = 0.6%

Bank C
B = -2.17
PD = 1.5%

Bank B
B = -2.58
PD = 0.5%

Uniform economy effect

Z, ρ

The economic cycle - Example of Z states – GLOBAL PORTFOLIO

- Raw data: default statistics d_t for the sector
- Estimation of Z_t and ρ by method of moments

$$\rho = \frac{\sigma^2}{1+\sigma^2} \quad \text{and} \quad Z_t = \frac{B-\Phi^{-1}(d_t)\sqrt{1-\rho}}{\sqrt{\rho}} = \frac{m-\Phi^{-1}(d_t)}{\sigma}$$

where m is the average and σ the standard deviation
of the time series $\Phi^{-1}(d_t)$
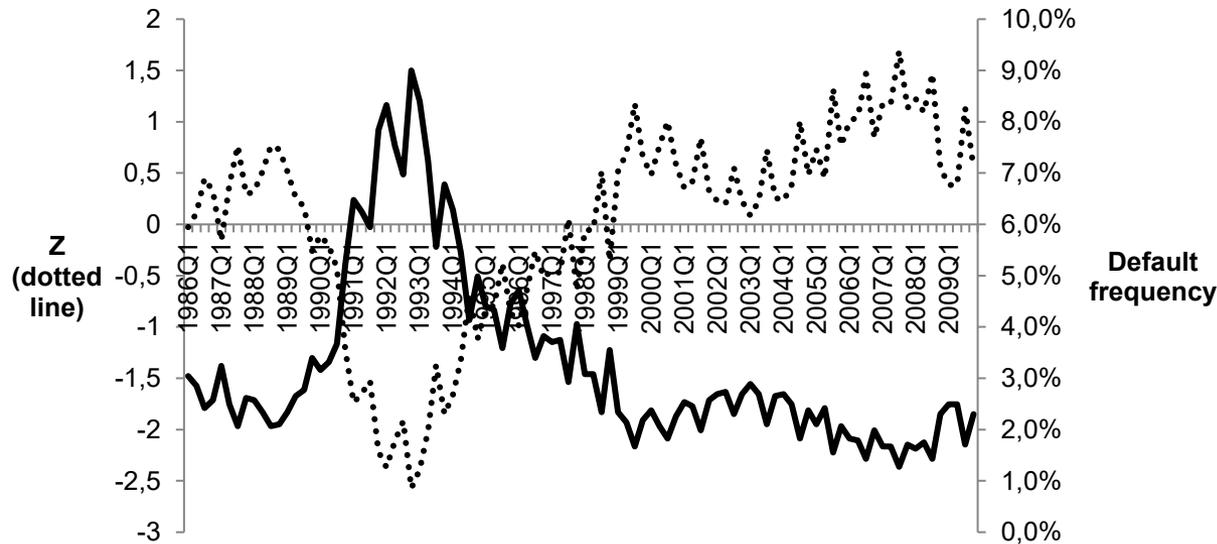
Examples of larger global portfolio

- Use of internal relationship between bankruptcy data and default rate to create final DF time series

- Instead of bankruptcy/defaults data the methodology could be run using external ratings PDs for specific segments

1. Estonia : **Centre of Registers and Information Systems** (2005Q1 and onwards)

2. Latvia**: Lursoft** (2005Q1 and onwards)

3. Lithuania: **Creditreform** (2005Q1 and onwards)

4. Sweden**: SCB (**1985Q1 and onwards), **UC (2001Q1** and onwards **)**

Calculating an obligor's TTC PD in the case of 100% PIT rating model

► Assuming the same average economy effect on all obligors, we have (as said)

$$\text{PIT\_PD}_i(Z) = P(X_i < B_i | Z) = \Phi\left(\frac{B_i - \sqrt{\rho} Z}{\sqrt{1-\rho}}\right)$$

► Averaging over all states of economy Z

$$\text{TTC\_PD}_i = \text{Average\_over\_Z}(\text{PIT\_PD}_i(Z)) = \text{Unconditional default probablity} = \Phi(B_i)$$
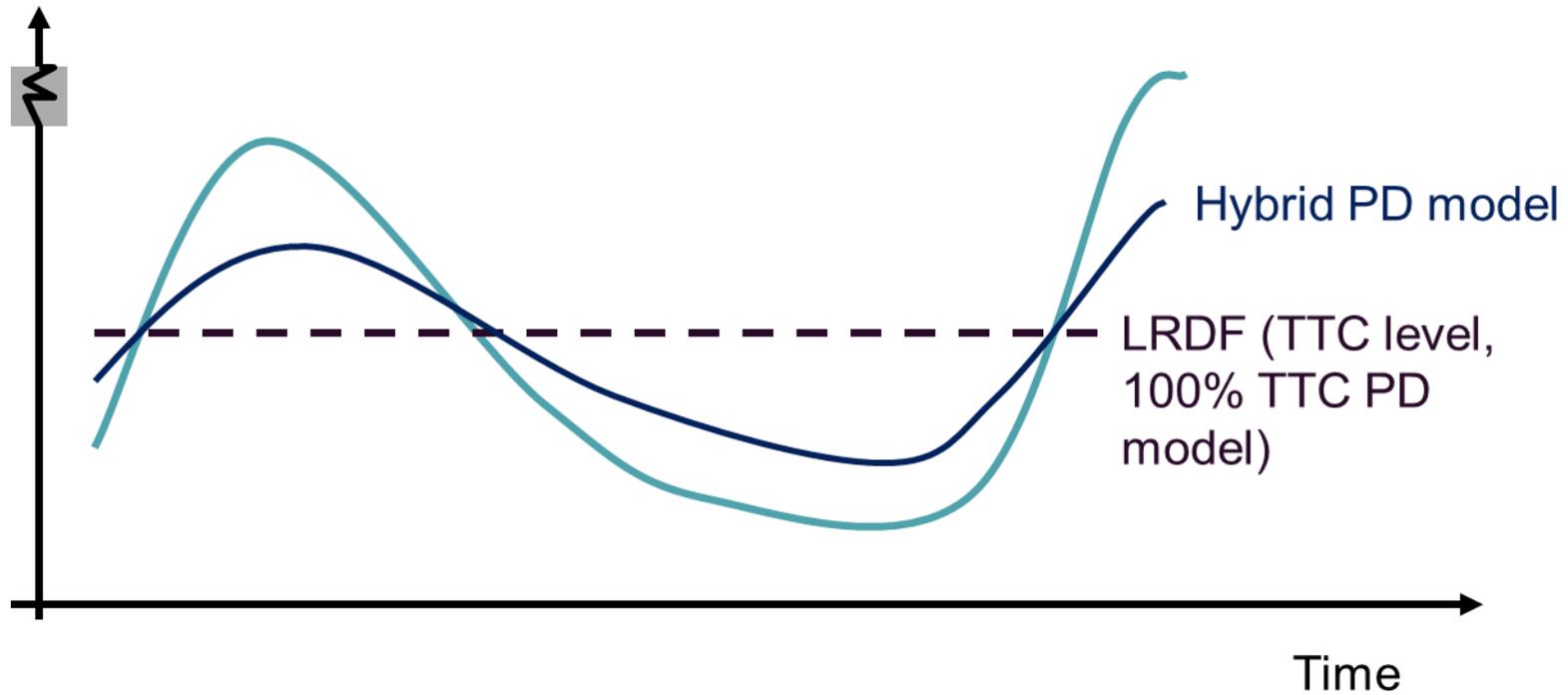
(classical result, exercise)

► To get TTC_PD_i we hence need to calculate the obligor specific B_i. By inverting the formula above and using the PIT_PD_i from our rating model:

$$B_i = \sqrt{\rho}\, Z_{\text{now}} + \sqrt{1-\rho}\,\Phi^{-1}(\text{PIT\_PD}_i),$$

and finally

$$\text{TTC\_PD}_i = \Phi\left[\sqrt{\rho}\, Z_{\text{now}} + \sqrt{1-\rho}\,\Phi^{-1}(\text{PIT\_PD}_i)\right]$$

How much is the PD model moving with the economic cycle?

Hybrid PD model

LRDF (TTC level, 100% TTC PD model)

Time

Calculating an obligor's TTC PD in the case of a hybrid rating model - "Modeling rating model behavior itself"

▶ Assuming a rating model which is partly PIT (say by a fraction α, $0 \leq \alpha \leq 1$) means that part of the economy effect is already averaged in rating model PD

$$\text{MODEL\_PD}_i =: \text{PD}_{i,\alpha} = \Phi\left[(B_i - \sqrt{\rho}\alpha Z)/\sqrt{1 - \rho\alpha^2}\right]$$

▶ For the limiting cases $\alpha = 0$ or $\alpha = 1$, we have TTC and PIT PD model respectively.

▶ Averaging over all states of economy Z gives the same result for TTC_PD as before:

$$\text{TTC\_PD}_i = \Phi(B_i),$$

▶ where similarly to the 100% PIT case

$$B_i = \sqrt{\rho}\,\alpha Z_{\text{now}} + \sqrt{1 - \rho\alpha^2}\Phi^{-1}(\text{PD}_{i,\alpha}),$$

▶ and hence

$$\text{TTC\_PD}_i = \Phi\left[\sqrt{\rho}\,\alpha Z_{\text{now}} + \sqrt{1 - \rho\alpha^2}\Phi^{-1}(\text{PD}_{i,\alpha})\right].$$

COMMENTS ON TTC_PD$_i$ and PIT_PD$_i$

$$\text{TTC\_PD}_i = \Phi(B_i) = \Phi\left[\sqrt{\rho}\,\alpha Z_{\text{now}} + \sqrt{1-\rho\alpha^2}\,\Phi^{-1}(\text{PD}_{i,\alpha})\right]$$

► In the ideal portfolio, with the same quality over time, the movement of the two terms within brackets is coordinated, for example, given upturn in the economy, Z increases and model PD decreases correspondingly, hence TTC_PD will be stable all the time.

► Sudden deterioration of client *i* in portfolio not connected to economy, but captured by rating model, will give rise to increase of second term, hence an increase of B_i, and of TTC_PD_i.

► Given Z$_{\text{now}}$ and model PD, for every quarter TTC_PD$_i$ can be calculated, while ρ as a slowly varying parameter, is estimated once a year.

$$\text{PIT\_PD}_i(z) = \Phi\left(\frac{B_i - \sqrt{\rho}z}{\sqrt{1-\rho}}\right) = \Phi\left(\frac{\sqrt{\rho}\,\alpha Z_{\text{now}} + \sqrt{1-\rho\alpha^2}\,\Phi^{-1}(\text{PD}_{i,\alpha}) - \sqrt{\rho}z}{\sqrt{1-\rho}}\right)$$

► PIT_PD can be calculated for any z, for instance in scenario testing

PIT PD and TTC PD, example
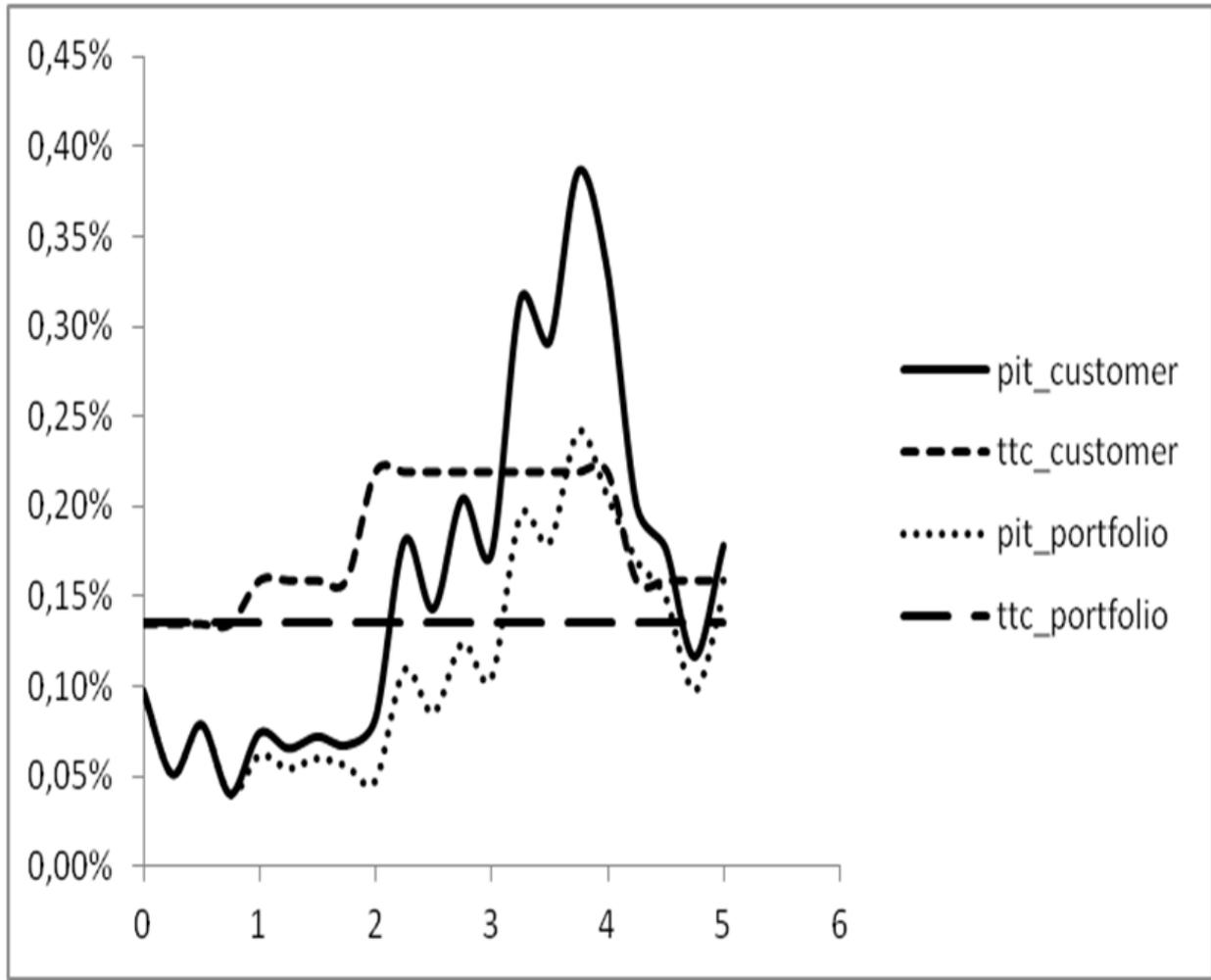
Method of estimation of degree of PIT (α) of PD models

► Assuming that portfolio composition is the same over time we have in two different points in time (with corresponding Z_1 and Z_2)

$$\Phi^{-1}\left(\mathrm{PD}_{i,\alpha}(Z_1)\right) = \frac{B_i - \sqrt{\rho}\alpha Z_1}{\sqrt{1 - \rho\alpha^2}}$$

$$\Phi^{-1}\left(\mathrm{PD}_{i,\alpha}(Z_2)\right) = \frac{B_i - \sqrt{\rho}\alpha Z_2}{\sqrt{1 - \rho\alpha^2}}$$

► Averaging over the portfolio and taking difference gives

$$\Delta \text{ Average\_portfolio } \left(\Phi^{-1}\left(\mathrm{PD}_{i,\alpha}(Z)\right)\right) = \frac{-\sqrt{\rho}\alpha \, \Delta Z}{\sqrt{1 - \rho\alpha^2}}$$

► From this equation the PIT degree α can be easily calculated. The result should be confirmed by the PD model owner and by review of the PIT degree of parameters of the corresponding regression model

Efficient regression-based evaluation of degree of PIT (α) of PD models

► Efficient regression-based evaluation of parametrization above suggested by Oeyen, B., & Salazar Celis, O., "On probability of default and its relation to observed default frequency and a common factor." Journal of Credit Risk, 15(3), 2019.

► The inverse of the normal distribution function of a calibration segments yearly PDs is regressed on the Z-factor:

$$\Phi^{-1}\left(Hybrid\ PD_t\right) = \gamma_{0,t} + \gamma_1 Z_t$$

► The obtained coefficient from the univariate regression (γ_1 (is then used to approximate the degree of PIT-ness expressed as alpha:

$$\alpha \approx \sqrt{\gamma_1{}^2/((1+\gamma_1{}^2)\rho)}$$

- Validation of B and ρ for global portfolio with new external data – once a year
- Validation of 100% PIT PD against actual DF data (usual Chi and Binomial tests)
- Validation of α – once a year (comparison of new point estimates with average α used)
- Other tests to fulfill regulatory demands, as, for example, simple correlation test
- Back testing of both TTC, PIT functionality as well as RWA for getting first FSA approval
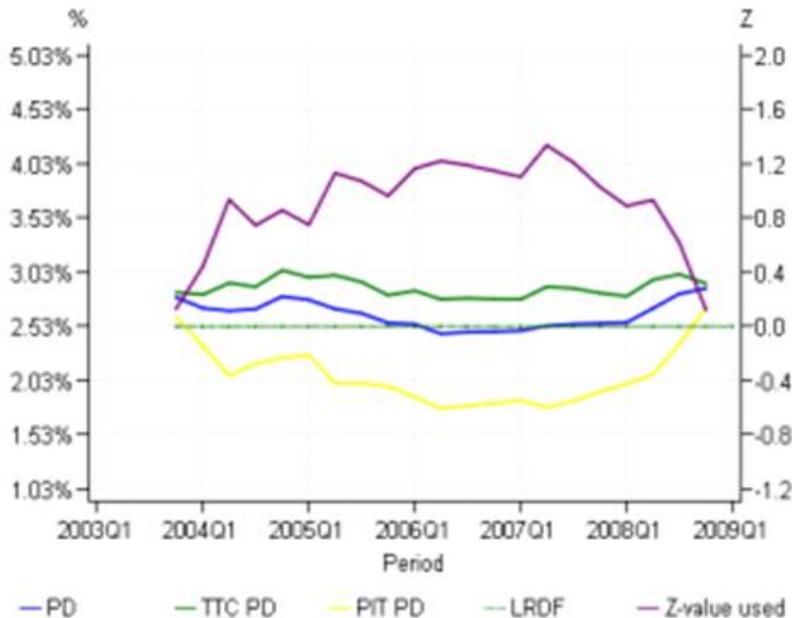
Algorithmic use: Annually/Quarterly updates

1. Annual Process

   i. Correlation is estimated annually, except in the case of soon downturn

   ii. Validation/estimation of PIT degree of each rating model and validation of 100% PIT PD against actual DF

2. Quarterly process

   i. Z value is calculated based on new external quarterly data

   ii. Seasonal adjustment for Z is done (if applicable)

   iii. TTC PD and RWA are calculated for each obligor rated in current quarter

   iv. Prediction of future values of Z are made, possible override of Z for steering.

   v. 100% PIT PD are calculated for each obligor

The output is two new PD

1. Pure PIT   Reflect current conditions
2. TTC Insensitive for cyclical movements, while sensitive for changes in risk profile

Internal (hybrid) Rating
Partly reflects current state of economic cycle. i.e. partly point-in-time

TTC PD
Output from the rating model is adjusted so that the corresponding PD estimate reflects an average economic state, i.e. is through-the-cycle

Legal capital Requirement
Scales up risk estimates reflecting an average economic state to the 99,9 % worst state of the economic cycle

PIT PD
"Pure" PIT PDs can be derived for each exposure reflecting the risk the coming year

Business steering
PIT PDs can improve business steering in all parts of the credit process

Algorithmic use of the method

|	Q1 2011	Q1 2012 (data)	Q1 2012 (steering)
Model PD	2%	2%	2%
Z	1.12	1.93	1
TTC PD	3.99%	5.82%	5.82%
RWA	128.6%	148.46%	148.46%
100% PIT PD	0.59%	0.33%	1.21%
PIT EL	0.27%	0.15%	0.54%
TTC EL	1.8%	2.62%	2.62%
RAROC 1y	17.38%	15.85%	13.19%
RAROC 5y	5.46%	-0.8%	-0.8%
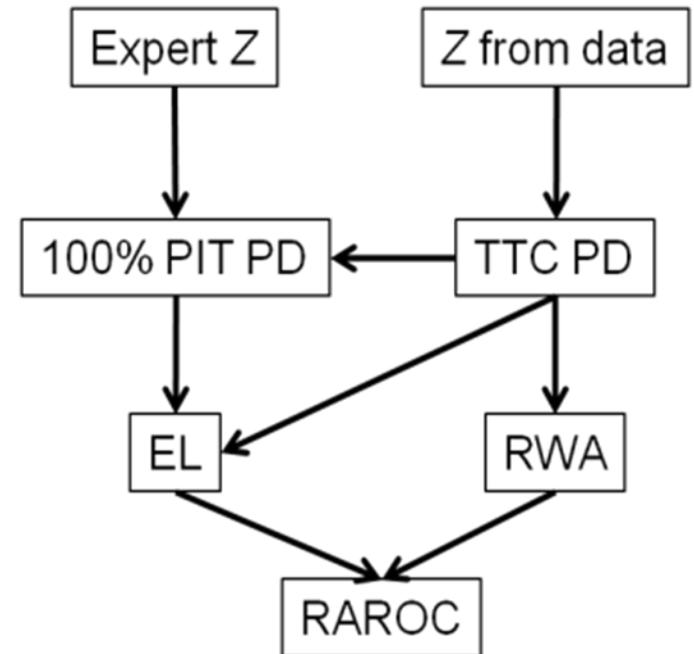
Extensions

► Until now no assumptions about Z dynamics in time are made.

► From realized time series we can see that Z values are strongly auto-correlated.

► If we have no underlying explaining variables for Z, one can model future Z pure statistically as AR process using historic Z values.

► If Z process can be explained by movement of macro economical variables a sensitivity type model can be developed.

► Both models can complement each other as macro model can be used for stress testing and ICAAP, where macroeconomic scenario is given, while for other situations pure statistical AR model can be used.

Extensions

- **Z as a economic cycle (credit cycle) process should exhibit two partially offsetting characteristics:**
  - Mean reversion – tends to trend up when low and trend down when high
  - Momentum – moves in the same direction as the previous year

- **Z is modeled as a second-order autoregressive process which has properties mentioned above:**
- $Z_t = k_1 * Z_{t-1} + k_2 * Z_{t-2} + e_t$
- **where e_t is normally distributed with 0 average.**
- **Degree of conservatism can be achieved by using upper value of 95% confidence interval for predicted Z instead of actual predicted value of Z**
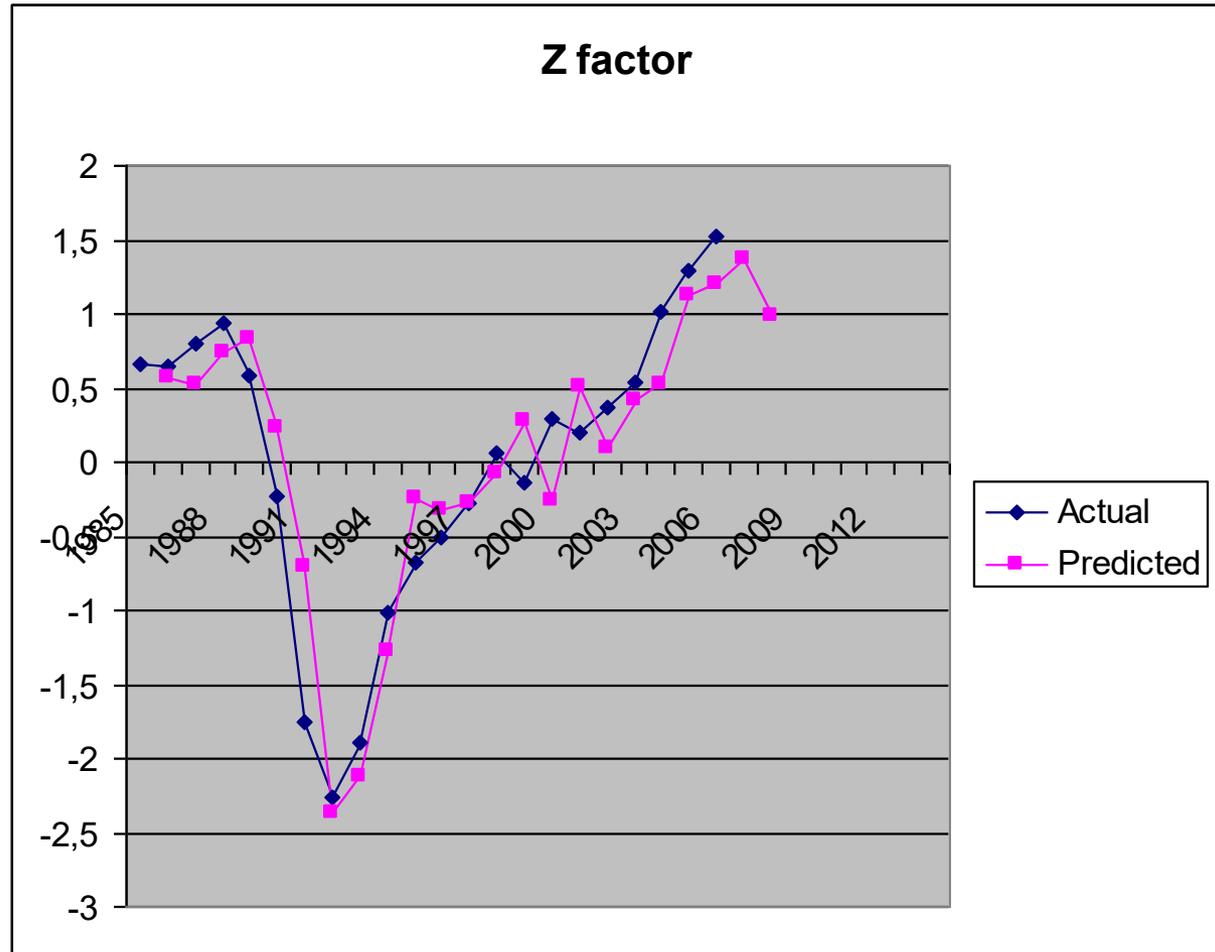
Z factor

Extensions

- **Makes PD (CL), RWA calculation for Pillars I and Pillars II on the same basis, Z factor is only predicted parameter for model**

$$\Delta Z = k_1 * \Delta \text{Short rate} + k_2 * \Delta \text{Unemployment} +$$
$$+ k_3 * \Delta \text{GDP} + k_4 * \Delta \text{Oil price} + k_5 * \Delta \text{SEKUSD} + \varepsilon$$

CONCLUSION: Advantages of this approach

► Basel IRB based methodology

► Transforms any PD value from a hybrid rating model to
    a) a TTC PD which is stable over time
    b) the corresponding 100 % PIT PD value

► Gives quantitative estimates of "point-in-time" degree for each hybrid rating model

► Can capture change in underlying risk characteristics of portfolio not connected to economic cycle

► Makes possible to run future economy stress scenarios per portfolio and per obligor's levels

► Use for IFRS9: Life time loan EL

► Use for stress testing: Z as model from macroeconomic factors

Application and usage of PIT-TTC PD methodology

Further methodolgy development after M. Carlehed & A. Petrov JRMV 2012

► EBA/GL/2017/16 on parameters estimation and TRIM have focus on the cyclicality as one of the sources of RWA variability

► IFRS9 and stress testing – need for PIT PDs, preferably based on IRB TTC PDs

► IIF working groups were trying to develop common industry standards in the PIT-TTC PD area

► Increased attention from regulators to the PD validation, monitoring of PD vs observed DF levels, with large focus on the cases where DF>TTC PD.

► Further development and improvement of M. Carlehed & A. Petrov approach:

  ► 1) Creating TTC rating/scoring grades and methodology for MOC estimation*, 2016

  ► 2) Efficient evaluation of parametrization in the M. Carlehed & A. Petrov approach**, 2019

  ► 3) The 2012 approach was also used for quantifying the effects of central bank and government interventions in the downturn. (for ex, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3926171), 2021

  ► 4) Attempts to use it for LGD and CCF (for ex, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3926176), 2021

  ► 4) The 2012 approach was used for climate risk stress testing, 2026,***

*Journal of Risk Model Validation, Volume 10/Number 2, Summer 2016, "A point-in-time–through-the-cycle approach to rating assignment and probability of default calibration", by Mark Rubtsov and Alexandre Petrov
**"On probability of default and its relation to actual default frequency and a common factor", by Brent Oeyen and Olivier Salazar Celis, Journal of Credit Risk, 15(3), 2019
***Calibrating credit risk parameters for climate stress testing " by Wojciech Starosta, Risk Management (2026) 28:1.

Regulators and industry has increasing focus on the topic of rating philosophies

(EBA/GL/2017/16):	Industry response/development:
" Institutions should **choose** an appropriate philosophy underlying the assignment of obligors or exposures to grades or pools (rating philosophy)"	**IIF**: "IRTF REVIEW: PIT and TTC Modelling POINT-IN-TIME AND THROUGH-THE-CYCLE MODELLING", IIF RISK WEIGHTED ASSETS TASK FORCE, October 2016
"…Institutions should **apply** the chosen rating philosophy **consistently over time**"	**IIF**: "Point-in-Time and Through-the-Cycle Modeling: Measures and Approaches for Harmonization", A report by the IIF RWA Task Force in cooperation with GCD, May 2018
"The rating philosophy should also be taken into account for **back testing** purposes."	

Rating philosophies can be big source of variability of RWA for the same portfolio.

Both PIT vs TTC PDs are required for key regulatory and BUs Objectives. They should be reconcilable. One source of the "truth" is needed

Model validation, PIT PDs

► It is more natural to validate PIT PD against realized DF for each year.

IFRS 9, PIT PDs

► For life time loan EL estimation PIT PDs for each year of the loan maturity are needed.

► 'If a bank uses IRB models for regulatory purposes, the bank may use the outputs from its IRB models as a starting point for calculating IFRS 9 PDs. However, the PDs from these IRB models may in some organisations be determined using a through the cycle (TTC) rating philosophy (or hybrid point-in-time-approach) or may include certain conservative adjustements (such as floors). Therefore, these PDs are appropriately adjusted if they are to be used for IFRS 9 purposes', GPPC, 2016, Par.2.32.2

RWA, TTC PDs

► For calculation of capital buffer against unexpected losses, the through-the-cycle PD (unconditional of the states of economic cycle, PD) should be used in the RWA formulas.

Stress testing, PIT PDs

► 'For the estimation of future losses in stress tests, institutions should, where appropriate, rely on credit risk parameters different from the ones applied in the calculation of capital requirements, which are usually through-the-cycle or hybrid parameters (a combination of through-the-cycle and point-in-time parameters) for PD and under downturn conditions for LGD. In particular, institutions should, where relevant, apply estimates based on point-in-time parameters in accordance with the severity of the scenario for ....' EBA – 'Methodology EU-wide Stress Test 2018', Par.114

TTC PDs are needed for RWA calculation, the PIT PDs are required for IFRS9, Stress Testing and Validation. For Reconciling IRB TTC PD values with IFRS 9 PIT PD values the same source of both is required

The measure of Pitness has to be defined, measured and tested

The **cyclicality of the rating system** can be interpreted as a measurement of its **degree of responsiveness to economic changes.**

- **Point-in-Time PiT 1-year PD** can be defined as an estimate of the probability of default (PD) within the following 1-year period based on the current state of the credit cycle. In case of a perfect PiT PD model, the PiT PD is expected to be equal to the **Actual Default Frequency (ADF)**.

- **Through-the-Cycle TTC 1-year PD** is an estimate of the average probability of default within the following 1-year period irrespective of the current state of the credit cycle. The **Long Run average of Actual Default Frequency (LRDF)** can be interpreted as a TTC PD.

- **Hybrid 1-year PD** is an estimate that lies somewhere in between a 100% PiT and a 100% TTC PD.

The rating models exhibit characteristics of both TTC and PiT Rating philosophies, hence are of a **Hybrid** nature.

PD

PiT PD (DF)

Downturn period of the cycle

Upturn period of the cycle

TTC PD (LRDF)
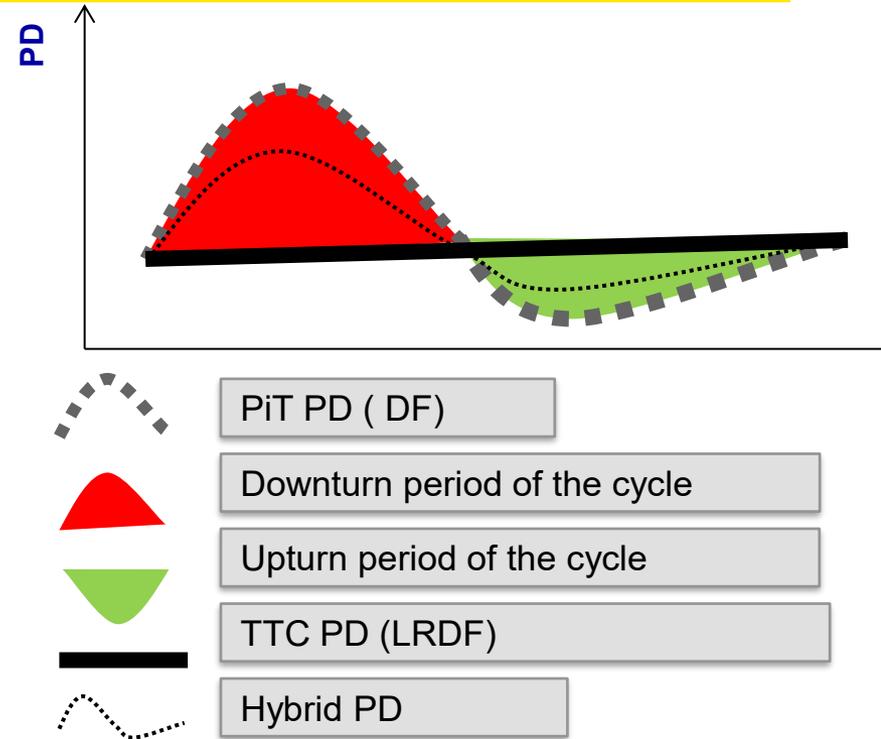
Hybrid PD

PiT Rating system

- Built on risk drivers that adjust rather quickly to the economic environment which results in high rating migration probabilities
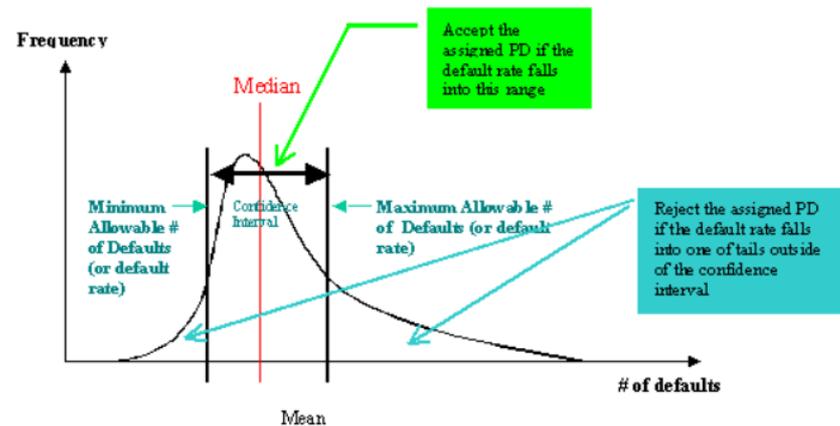
Hybrid Rating system

Shows characteristics of both PiT and TTC.

TTC Rating system

- Built on risk drivers that are more independent to the economic environment which results in low rating migration probabilities

Fulfilling Basel regulatory testing requirements with the predictive power of PD models

► PIT testing: actual vs expected

► Chi-Square test with non-zero asset correlation assumption - allows to pass and explain volatility of DR

► TTC testing - Long run average default rate (LRADR) test for new observed DR

► MOC simulation testing for calibration of LDP," A Proposal for Conservative Estimation of Default Probabilities", by Nathanaël Benjamin, Alan Cathcart and Kevin Ryan, Financial Services Authority, 2006

► Pitnes/cyclicality testing can be done with confidence intervals based on above

See also SYSTEMATIC BACKTESTING OF PROBABILITY OF DEFAULT MODELS WITH REGULATORY DATA, EBA STAFF PAPER SERIES, by Simone Casellina, Gaetano Chionsini, Raphael M. Kopp, and Maroua Riabi, 2025

IRB: MoC C quantification for Low Default Portfolios with Merton based BCR (ref1) simulation approach

BCR description:

1) Input parameters in original paper:
   Confidence level: for example γ = 75%
   Asset correlation: for example ρ = 12%
   Year-to-year correlation: for example θ = 30%
   Number of simulated samples N= 1000000

1) Input Data:
   Number of observed years T
   Number of observations n
   Number of observed defaults r

1) Output:
   LRADR_est given confidence level, meaning that for ex with 75% probability real value of true LRADR is less than LRADR_est

BCR estimation procedure:

a) Simulate values of the standard normally distributed systematic risk factor which consist of independent draws of the T- year sequence S1,…,ST, given inter- year correlation structure determined by neighbouring years correlation of systemic factor equal to θ, so that the correlation between Years i and j is $θ^{|i-j|}$.

b) Solve numerically equation to get p=LRADR_est:

$$1 - γ = \sum_{k=0}^{r} \binom{n}{k} \, Average\left((π(S1,…,ST))^{k} * (1 - π(S1,…,ST))^{n-k}\right)$$

where

$$π(S1,…,ST) = 1 - \prod(t=1 \text{ to } T)\left(1 - Φ\left(\frac{Φ^{-1}(p) - \sqrt{ρ}\, St}{\sqrt{1-ρ}}\right)\right)$$

Procedure should be run with different parametrisation:

a) Different confidence levels
b) **Asset correlation can be set initially as in Basel for corporates to 12%. However actual values for Scandinavia and Germany can be lower for corporates – at the level about 5-8%. Estimation of actual asset correlation could be done using DR statistics, see ref (2,3).**
c) Year-to-year correlation – should be estimated by using DR statistics through calculation of Year-to-year correlation of Z factor in ref (2).
d) N should be high enough, so convergence of results is achieved

[1]Low Default Portfolios: A Proposal for Conservative Estimation of Default Probabilities, by Nathanaël Benjamin, Alan Cathcart and Kevin Ryan, Financial Services Authority, 2006

[2]A methodology for PIT-TTC PD decomposition in credit risk classification systems, by Magnus Carlehed and Alexandre Petrov Journal of Risk Model Validation, Volume 6/Number 3, Fall 2012 (1–23)

[3]ASSET CORRELATIONS IN CREDIT RISK, AN EMPIRICAL STUDY WITH GCD DATA, 2025, by Bromma, Plaetevoet, Fleckenstein, Guttierez, Petrov, https://globalcreditdata.org/wp-content/uploads/2025/09/gcd-asset-correlation-report-2025.pdf

Similar ideas applied for back testing of Non-LDP, see SYSTEMATIC BACKTESTING OF PROBABILITY OF DEFAULT MODELS WITH REGULATORY DATA, EBA STAFF PAPER SERIES, by Simone Casellina, Gaetano Chionsini, Raphael M. Kopp, and Maroua Riabi, 2025

Stress Testing: The effect of the PiTness on Capital Requirements: Scenario Illustration

	Upturn	Normal	Downturn
RWA (Hybrid PD)	74%	90%	106%
RWA (TTC PD)	92%	92%	92%
RWA (PiT PD)	53%	87%	119%
TTC PD	1,50%	1,50%	1,50%
ADF	0,46%	1,30%	3,22%
Hybrid PD	0,89%	1,45%	2,28%
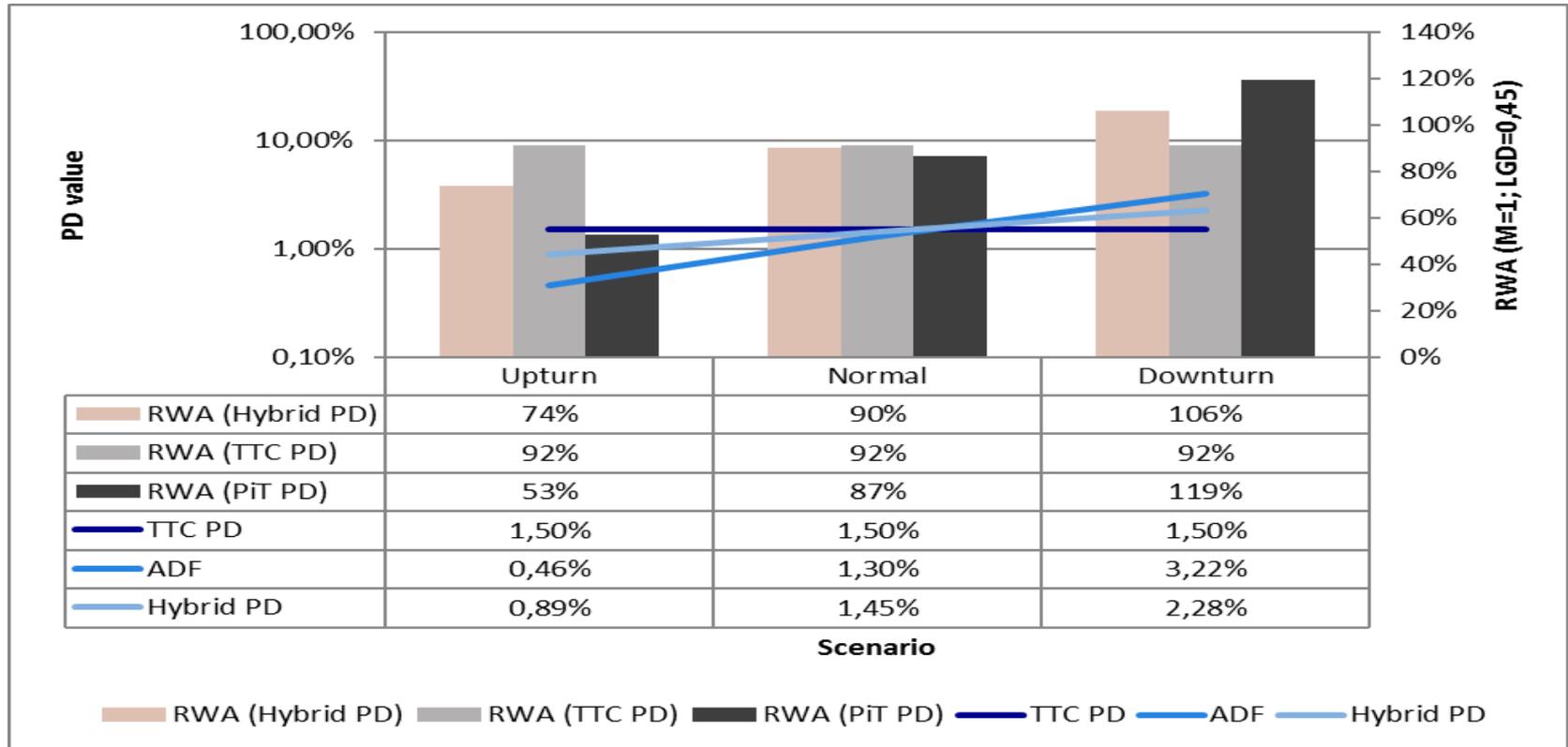
During downturn periods, Capital Requirements for the same portfolio are highly impacted by the PiTness of PD values

Approach to model Hybrid PDs, 100% PIT PD (DR), TTC PDs combining macro economical scenario and company based CO2 tax impact, based on M. Carlehed & A. Petrov*

$$p_{i,\alpha}(z) = \Phi\left(\frac{B_i - \sqrt{\rho}\alpha z}{\sqrt{1 - \rho\alpha^2}}\right)$$

Rating model gives customer-specific Hybrid PD

Bankruptcy statistics/Ratings provide estimate of the business cycle

'Portfolio with stable risk profile' provides measure of PIT-ness, alpha

Portfolio-level default rates provide the rest of the estimates

TTC PD

PIT PD Y1

PIT PD Y2

...

Macroeconomic forecast
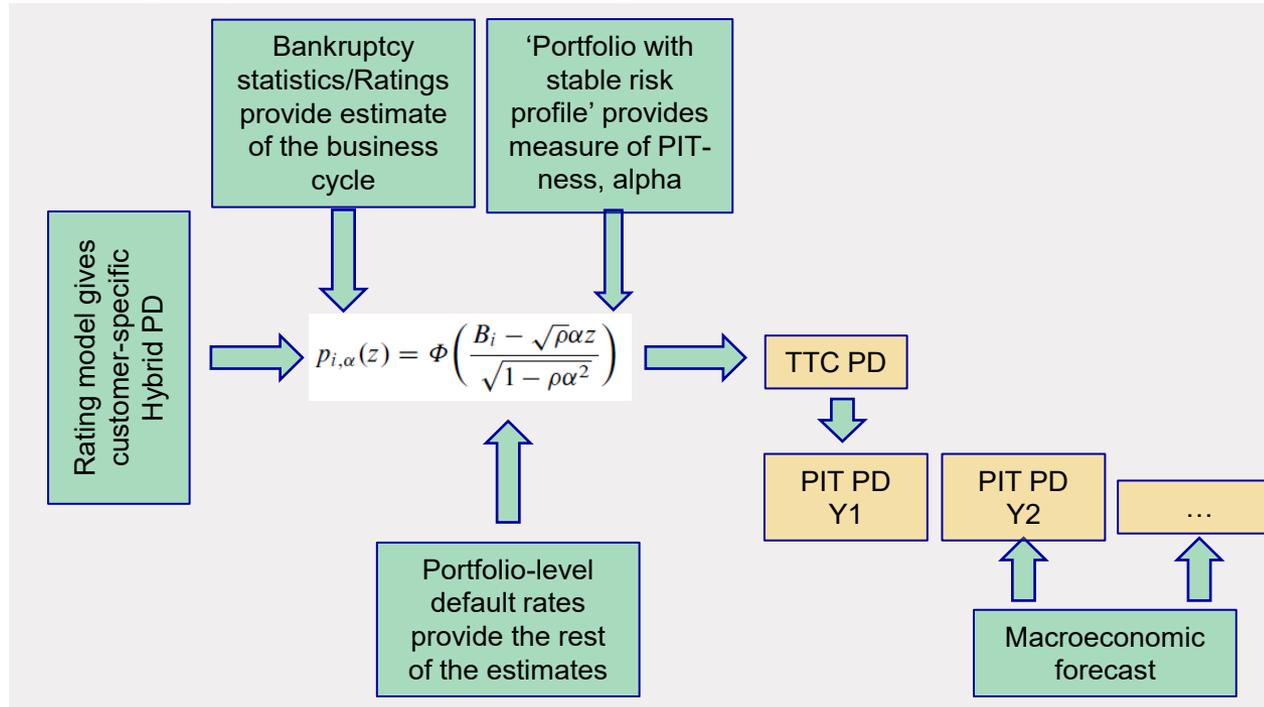
* High level overview and applications of methodology: SAS Insights, 2022, *https://www.sas.com/en_us/insights/articles/risk-fraud/understanding-capital-requirements-in-light-of-basel-iv.html*

* Details: Journal of Risk Model Validation, Volume 6/Number 3, Fall 2012 (1–23), "A methodology for PIT-TTC PD decomposition in credit risk classification systems", by Magnus Carlehed and Alexandre Petrov, *JRMV the Most cited Paper*

Calculating an new obligor's PDs for hybrid rating model in presence of climate risk

► Assuming a rating model which is partly PIT (say by a fraction α, $0 \le \alpha \le 1$) means that part of the economy effect is already averaged in rating model PD

$$\mathrm{MODEL\_PD}_i =: \mathrm{PD}_{i,\alpha} = \Phi\left[(B_i - \sqrt{\rho}\alpha Z)/\sqrt{1 - \rho\alpha^2}\right]$$

► For the limiting cases $\alpha = 0$ or $\alpha = 1$, we have TTC and PIT PD model respectively.

► Averaging over all states of economy Z gives the same result for TTC_PD:
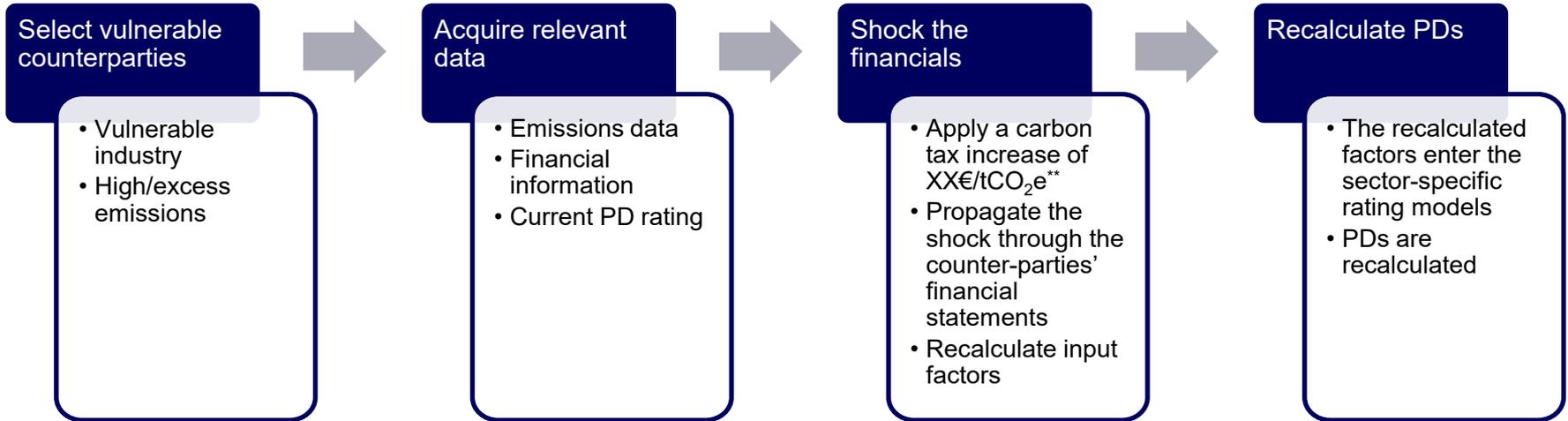
$$\mathrm{TTC\_PD}_i = \Phi(B_i),$$

► so

$$B_{i,climate} = \sqrt{\rho}\,\alpha Z_{\mathrm{now}} + \sqrt{1 - \rho\alpha^2}\,\Phi^{-1}(\mathrm{PD}_{i,\alpha,climate}).$$

► where $\mathrm{PD}_{i,\alpha,climate}$ is stressed $\mathrm{PD}_{i,\alpha}$ through PD model risk factors impacted by climate scenario.

Calculation of $\mathrm{PD}_{i,\alpha,climate}$

Select vulnerable counterparties

- Vulnerable industry
- High/excess emissions

Acquire relevant data

- Emissions data
- Financial information
- Current PD rating

Shock the financials

- Apply a carbon tax increase of XX€/tCO$_2$e[**]
- Propagate the shock through the counter-parties' financial statements
- Recalculate input factors

Recalculate PDs

- The recalculated factors enter the sector-specific rating models
- PDs are recalculated

Combining micro customer level impact with macro portfolio level impact, in the correct modelling way, using Carlehed Petrov 2012 approach (examples are climate risk , covid, state interventions, geopolitical impact, etc)

Stress testing example: Combining bottom-up climate impact with top-down macro economical impact

1. Having fully parametrized M. Carlehed & A. Petrov, 2012 approach for given portfolio in place

2. Identify customers impacted by climate risk

3. For these customers stress rating model risk factors with bottom-up climate impact, for ex through increase of carbon tax

4. Get new hybrid PD after previous step on customer level

5. Recalculate new climate stressed TTC PD (Bi) for these customers given other input as today, Z etc

6. Use new TTC PD (Bi) for these customers in the macro economic scenario: for PIT PD calculations (for EL) and for hybrid PD calculations (for RWA) in the given macroeconomic stress scenario, Z =function (GDP, …)

More details of usage of methodology by Carlehed Petrov 2012 for integrating climate risk in stress testing and IFRS9 can be found in recent article "Calibrating credit risk parameters for climate stress testing " by Wojciech Starosta, Risk Management (2026) 28:1.

Climate risk in credit risk models in Banks vs. Climate Reality – with Alexander Petrov Podcast:
https://m.youtube.com/watch?v=sXnPuBdXJas

Cyclicality/Rating philosophy as a source of regulatory MoC/add-on

Typical issues banks need to explain to regulators:

1) Why observed DRs are higher than PD for consecutive periods?

Risk of possible regulatory action: Introduce MoC(add-on) based on deference among observed DRs and PDs
Bank need to prove that there is no deterioration of idiosyncratic risk, deviations are of cyclical nature, as well as these deviations are in line with chosen rating philosophy and pitnes.

2) Do the observed DR values correspond to the chosen level of LRADR used for model calibration?

Risk of possible regulatory action: Force bank to increase LRADR
Bank need to prove that observed DR values are inside confidence interval of allowed DR variations for a given level of LRADR.
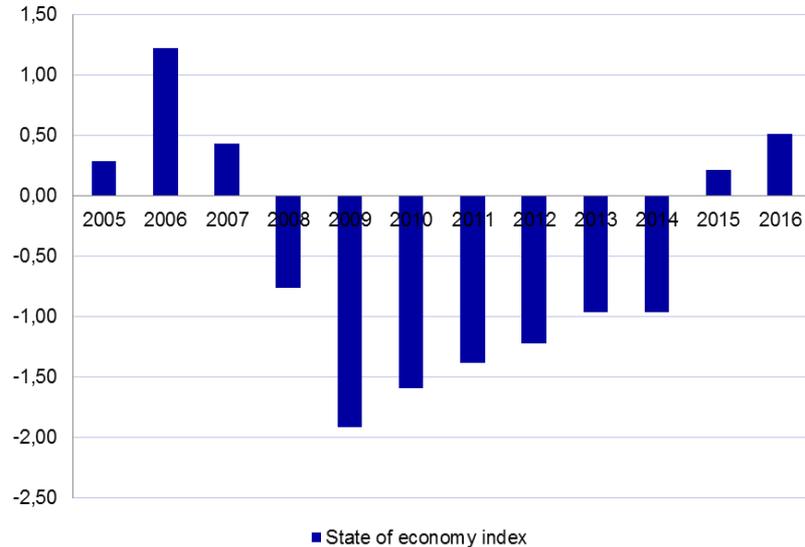
3) Do the variations of PD correspond to the chosen rating philosophy/pitness?

Risk of possible regulatory action: MoC(add-on) for rating philosophy mismatch.
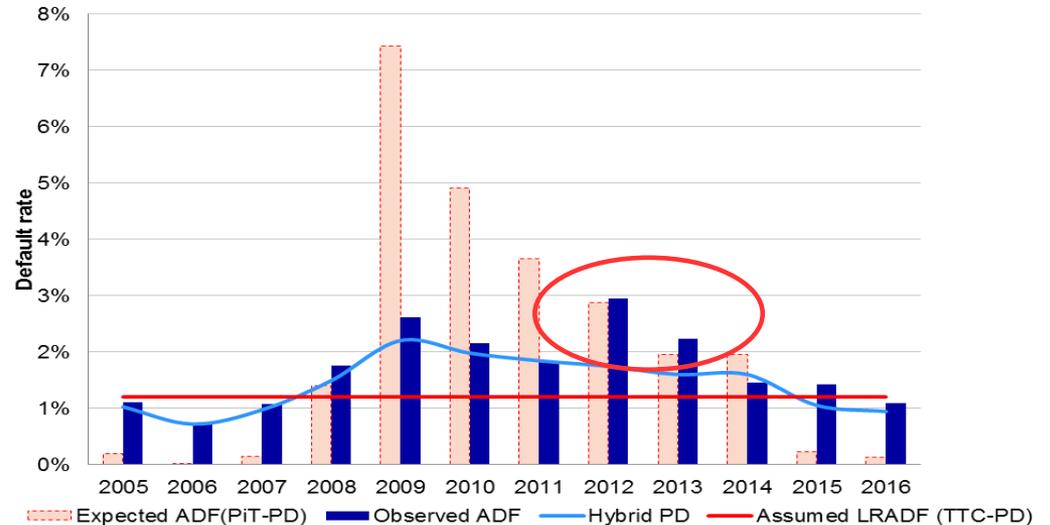Bank need to measure pitness and prove that its according to expectation and regulatory approval.

Practical example: Actual DFs vs PD values - discussion with regulator

State of economy index

DFs, TTC and Hybrid PDs overtime

Legend: Expected ADF(PiT-PD) · Observed ADF · Hybrid PD · Assumed LRADF (TTC-PD)

Banks should have clear view on the degree of cyclicality of PDs as well as on the effect of the economic cycle on both PDs and portfolio DFs

Even having almost TTC PD calibration banks should anyway explain periods with actual DFs higher than PDs for the FSA, and provide a proof that this is due to the economic cycle effects and not due to the deterioration of the portfolio itself

8 keyways the industry is using the PIT-TTC methodology

Details can be found on SAS white paper, 2022: **Understanding capital requirements in light of Basel IV | SAS**

- Establishing a fair dialogue with regulators about portfolio risk versus required capital levels

- Fulfilling Basel regulatory testing requirements with the predictive power of PD models

- Reconciling IRB TTC PD values with IFRS 9 PIT PD values

- Fulfilling Basel regulatory requirements for cyclicality or PIT-ness assessments for PD models

- Incorporating stress testing based on IRB PD models

- Having a view on increased capital requirements in a downturn

- Using IRB-based pricing as part of IRB use case test

- Allowing a separate change of PD at the obligor level

https://www.sas.com/en_us/insights/articles/risk-fraud/understanding-capital-requirements-in-light-of-basel-iv.html

Thank you!

Q & A